Solutions for Tutorial 9 (Petri Nets)

1. Give the Matrices \( E^+ \), \( E^- \) and \( m_0 \) of the Petri Net below.
2. Draw the reachability graph of the Petri Net below.

3. Draw the Petri Net with following Matrices:

\[
E^+ = \begin{pmatrix}
2 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\quad
E^- = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\quad
m_0 = \begin{pmatrix}
1 \\
1 \\
0 \\
0 \\
1
\end{pmatrix}
\]

Draw the Petri Net after the first switch, concerning the final state with only one token remaining
Solutions

1. \[ E^+ = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad E^- = \begin{pmatrix} 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad m_0 = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \]

2. The reachability graph looks like this:

```
(2 0 0 0)     (1 1 0 0)     (1 0 0 1)
|      |      |      |
T_5    T_3    T_1    T_5
|      |      |      |
(0 0 1 0)  (0 2 0 0)  (0 2 0 0)
|      |
T_2    T_1
|      |
(0 0 1 1)
|      |
T_4    T_1
|      |
(0 0 0 2)
```
3. The Petri net is illustrated in figure 1. If we want only one token left in the final state, T₃ has to fire first.

Otherwise if T₂ fires first, we have two tokens in P₁ and one in P₅, after T₁ fires we reach the final state with one token in P₃ and one in P₅.