Bankruptcy Cascades in Interbank Markets

Structure

- Overview
- Model
- Simulations
- Conclusion
Overview

Demand | Firms | Banks

Time $t$

0 1 2 3 4 5 6 7 8 9 10
Firms

Demand:
\[ D_{f,t} = D_{f,t-1}(1 + g_0 + \varepsilon_{f,t}) \]
«Demand before; increased by a given percentage varied by a normal distribution»

Expected Demand:
\[ E(D_{f,t}) = D_{f,t-1}(1 + g_0) \]
«Demand before; increased by a given percentage»

Production:
\[ Y_{f,t} = \phi K_{f,t-1} \]
«Maximum production = production cost times capital»
Firms

Needed loan:
\[ L_{f,t} = \max \left\{ \frac{E(D_{f,t})}{\phi} - Y_{f,t} , 0 \right\} \]
«Money needed to meet expected demand minus own capital»

Demanded loan:
\[ L_{f,t}^d = \alpha \left( 1 - \frac{L_{f,t}}{E(\pi_{f,t+1})} \right) L_{f,t} \]
«Money asked for depending on risk aversion and expected debt/profit ratio»

Debt Commitments:
\[ \overline{L}_{f,t} = \frac{1}{t} \sum_1^t (1 + i_{f,b}^t) L_{f,t}^d \]
«Average money they give banks»

Expected profit:
\[ E(\pi_{f,t+1}) \]
«Money they expect to get»
Firms

Profit:
\[ \pi_{f,t} = \min\{D_{f,t}, Y_{f,t}\} - L_{f,t} \]
«Money they get from sells minus money they give banks»

Capital:
\[ K_{f,t} = K_{f,t-1} + \pi_{f,t} + L_{f,t}^d \]
«Money before plus profit plus loans»
Banks

Offered interest rate:

\[ i_{t}^{f,b} = \bar{i} + \left( \frac{L_{f,t}^{d}}{S_{b,t}} \right)^{\alpha} \]

«Smaller interest rate when bank has more liquidity»

Probability of loan:

\[ c_{t}^{f,b} = 1 - \beta \left( \frac{L_{f,t}^{d}}{S_{b,t}} \right)^{\sigma} \]

«Higher \( \beta \) means less granted loans»

Probability of loan:

\[ c_{t}^{f,b} = 1 - \beta \left( \frac{L_{f,t}^{d}}{S_{b,t}} \right)^{\sigma} \]

«Higher \( \beta \) means less granted loans»
Banks

1000$?
Banks

1000$? 2%

4%

5%
Banks

1000$?
2%

700$

4%

5%
Banks

1000$?  2%

700$

300$?
Banks

1000$ 
2%

300$ 
1%
Banks

Liquidity:

\[ S_{b,t} = S_{b,t-1} - \sum_f L_{f,t}^d + \frac{1}{\tau} \sum_{f,t-\tau<t'<t} L_{f,t}^d (1 + i_{t',b}^f) + I_{b,t} \]

«Liquidity before minus given loans plus returned money from bank plus/minus lending from/to other banks»

Loan from bank j to bank i:

\[ I_{b,i,t} = -\frac{1}{\tau} \sum_{b,j,t-\tau<t'<t} L_{b,j,t'}^d (1 + i_{t',b}^{b,j}) + \sum L_{b,j,t}^d \]

«minus average returned money plus new lend money»
Banks

Profit:
\[ \pi_{b,t} = \frac{1}{\tau} \sum_{f,t-\tau<t'<t} L_{f,t}^d i_{f,b}^t - \sum_{t'} \omega_i^f L_{f,t'}^d + P_{b,t} \]
«interest paid by firms minus lost loans (firm went bankrupt) plus interbank credit»

Interbank profit:
\[ P_{b,j,t} = \frac{1}{\tau} \sum_{b_i,t-\tau<t'<t} L_{f,t}^d i_{b_i,b}^t - \sum_{b_{i'}} \omega_i^{b'} L_{b_i,t'}^d \]
«interest paid by banks minus lost loans»
Rules

- Not enough money to pay interest => Bankrupt
- More new firms enter when interest rate is low
- No banks enter
- Linkage between banks can be fixed
Figure 1. Evolution of the aggregate output (left side) and growth rates of the aggregate output (right side), as a function of time.
Figure 2. Time evolution of firm bankruptcies (left side) and decumulative distribution function of failed firms’ size (right side).
Figure 3. Time series of granted loan (left side) and the inverse of the firms’ leverage (right side).
Figure 4. Decumulative distribution function of firm sizes (left side) and bank sizes (right side).
Variation

- Bank reserves ($\beta$)

  Probability of loan:

  \[ c_{t}^{f,b} = 1 - \beta \left( \frac{L_{f,t}^{d}}{S_{b,t}} \right)^{\sigma} \]

  «Higher $\beta$ means less granted loans»

- Interbank linkage
Figure 5. Time evolution of the number of surviving banks for different levels of reserve ratios: 0.1 (solid line), 0.5 (dotted line) and 0.9 (long dashed line) (left side).

Average bank’s leverage over time and simulation as a function of $\beta$ (center)

Average output growth rate over time and simulation as a function of $\beta$ (right side)
Figure 6. Time evolution of the number of surviving banks with for different interbank linkages: 1 (solid line), 2 (dotted line), 5 (dashed line), 10 (long dashed line), 49 (dot-dashed line) (left side).

Average number of surviving banks as a function of x (center)
Average absolute slope of the curve representing the number of surviving banks (right side) as a function of x.
Figure 7. Average bank’s leverage (left side). Average output growth rate (right side), over time and simulation as a function of $x$. 
Figure 8. Size of the largest bankruptcy cascades, which are connected by bad debits for a bank market of size 50, determined from 100 simulations for interbank linkages of 1, 5, 10, and 49.
Figure 9. Decumulative distribution function of failed bank’s size $S$, for 1 (solid line), 2 (dotted line), 5 (dashed line), 10 (long dashed line) and 49 (dot-dashed line) (left side).
Conclusions

- Higher reserves leads to higher stability
- But also a lower growth

- Higher bank linkages leads to higher system risk
- And not to a lower growth

- Heterogeneity contributes to instability
Thanks!

Questions?