Mathematical Sociology, Agent-Based Modeling and Artificial Societies.

Prof. Dr. Dirk Helbing and Team

Chair of Sociology, in particular of Modeling and Simulation

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Opinion Dynamics
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Overview

1. Introduction (consensus, persistent opinion diversity and polarization)
2. Modeling opinion dynamics
3. Main ideas and references
Motivation

A group of interacting agents involved in a process of opinion formation

Possible outcomes: Consensus, Polarization or opinion fragmentation.

**Motivation**: High social impact of the different outcomes.
Introduction

Consensus, opinion diversity and polarization

Consensus

Studies on consensus formation dealing with:

- Time needed to reach consensus.
- The role of social networks.
- How initially minority opinions can become the consensus ones.
Persistent opinion diversity

"If people tend to become more alike in their beliefs, attitudes and behaviour when they interact, why do not all such differences eventually disappear?" (Axelrod, 1997)

Previously identified mechanisms:
- Social differentiation
- Drift
- Specialization
- Changing environment and technology

But, what about the interindividual communication? (the role of homophily).
Polarization

- Public opinion formation: Extreme opinions can polarize moderate ones.
- Two cases (Mäs and Flache, 2008): bi-polarization (two extremes) and polarization (one extreme)
  - bi-polarization:
    - Observed regarding polemic issues (abortion, Iraq war..)
    - Influence on group dynamics of work teams.
  - polarization:
    - Some have pointed religious beliefs in Islamic world as an example
    - Replicated in laboratory experiments
Impact of opinion polarization and high homophily on social cohesion.
Modeling opinion dynamics

- **Goal**: How macrophenomena can emerge from interactions among individuals

- More concretely, explain the observed outcomes (consensus, polarization and diversity persistence) from mechanisms based on social influence and homophily.
Examples of models

- Opinion dynamics and bounded confidence (Hegselmann and Krause, 2002).

- Local Convergence and Global Diversity (Axelrod, 1997).
Bounded confidence

(Hegselmann and Krause, 2002) (see (Lorenz, 2007) for a review of models)

Classical models of continuous opinion dynamics based on static interindividual influence $\Rightarrow$ consensus.

Solution: Let the weights change over time.

Agents consider distinct opinions, but only to a limited level (bounded confidence).
Continuous opinion models

- A population of $n$ agents

- $x_i(t)$: Agent’s $i$ opinion at time $t$ (represented by a real number).

- $a_{ij}$: Weight of agent’s $j$ influence over agent’s $i$ opinion. $a_{ij} > 0$ and $a_{i1} + a_{i2} + \ldots + a_{in} = 1$

- Then: $x_i(t + 1) = a_{i1}x_1(t) + a_{i2}x_2(t) + \ldots + a_{in}x_n(t)$

- Classical approach (Abelson 1964)(DeGroot 1974) (Lehrer 1975): $a_{ij}$ fixed $\Rightarrow$ opinion uniformity (unless completely ’disconnected’ agents)
Bounded Confidence

At each time step, agent $i$ only takes into account opinions which differ from it's own less than a certain confidence level $\epsilon_i$:

$$I(i, x) = \{1 \leq j \leq n | |x_i - x_j| \leq \epsilon_i\}$$

Then:

$$a_{ij} = \begin{cases} 0 & \text{if } j \notin I(i, x) \\ \frac{1}{|l(i, x)|} & \text{Otherwise} \end{cases}$$

And:

$$x_i(t + 1) = \frac{1}{|l(i, x)|} \sum_{j \in I(i, x(t))} x_j(t)$$
Symmetric confidence

$$\epsilon_i = \epsilon \text{ and } [-\epsilon, +\epsilon]$$

- $$\epsilon = 0.01$$
- $$\epsilon = 0.15$$
- $$\epsilon = 0.25$$
Opinion INDependent asymmetric confidence

\[
\left[-\epsilon_l, +\epsilon_r\right]
\]

\[
\begin{align*}
\epsilon_l &= 0.02 \\
\epsilon_r &= 0.04 \\
\epsilon_l &= 0.03 \\
\epsilon_r &= 0.15 \\
\epsilon_l &= 0.10 \\
\epsilon_r &= 0.25
\end{align*}
\]
Opinion DEPendent asymmetric confidence

- $m$ controls the bias in the model
- $\beta_r(x) = mx + \frac{1-m}{2}$ and $\beta_l(x) + \beta_r(x) = 1$
- Then: $\epsilon_l = \epsilon\beta_l(x)$ and $\epsilon_r = \epsilon\beta_r(x)$
Opinion DEPendent asymmetric confidence (2)

\[ \epsilon = 0.6 \quad \text{and} \quad m = 0.99 \]

\[ \epsilon = 0.6 \quad \text{and} \quad m = 0, 0.25, 0.5, 0.75 \]
Disadvantages of the model

- No real emergence of extreme opinions from moderate ones (due to averaging)
- Too much ’built in’ (the model is reformulated in order to get certain desired outcomes)
Local Convergence and Global Diversity

- (Axelrod, 1997)

- High impact in different fields (sociology, physics..).

- Why social influence does not (always) lead to opinion homogeneity?
The model

Two main assumptions:

- “people are more likely to interact with others who share many of their cultural attributes” - homophily
- “interactions between two people tend to increase the number of attributes they share” - social influence
**Initial conditions**

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>A Typical Initial Set of Cultures</th>
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<tbody>
<tr>
<td>74741</td>
<td>87254</td>
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<td>01948</td>
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<td>55285</td>
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</tbody>
</table>

| 82330   | 17993                            |
| 67730   | 89130                            |
| 42628   | 86636                            |
| 51585   | 84468                            |
| 72031   | 19856                            |
| 48416   | 85455                            |
| 34235   | 45602                            |
| 77404   | 17043                            |
| 80173   | 81447                            |
| 66329   | 30462                            |

| 22978   | 82762                            |
| 34210   | 85403                            |
| 27405   | 39747                            |
| 18122   | 60094                            |
| 08071   | 97744                            |
| 55455   | 88600                            |
| 39891   | 84866                            |
| 39238   | 81454                            |
| 22884   | 58260                            |
| 30462   | 36729                            |

| 87476   | 26757                            |
| 69411   | 81677                            |
| 97450   | 71833                            |
| 71819   | 51912                            |
| 42533   | 33723                            |
| 96775   | 86714                            |
| 78008   | 27136                            |
| 74576   | 41924                            |
| 53436   | 13623                            |
| 43987   | 43378                            |

| 99313   | 32009                            |
| 06789   | 24042                            |
| 07192   | 87426                            |
| 32095   | 11318                            |
| 03847   |                                 |
| 02932   |                                 |
| 50153   |                                 |
| 43987   |                                 |
| 46585   | 47330                            |

**NOTE:** The underlined site and the site to its south share traits for two of the five cultural features, making a cultural similarity of 40%.
Dynamics

Each time step:

1. Select one agent \( i \) at random
2. Choose one of its neighbours \( j \) also at random
3. Calculate similarity among their features \( s \)
4. Agents \( i \) and \( j \) interact with a probability equal to \( s \). If interaction:
   - Select one feature where \( i \) and \( j \) differ.
   - Let \( i \) adopt \( j \)'s value at that feature
Example

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NOTE: The underlined site and the site to its south share traits for two of the five cultural features, making a cultural similarity of 40%.
Figure 1: Map of Cultural Similarities

NOTE: Cultural similarity between adjacent sites is coded as black ≤ 20%, dark gray = 40%, gray = 60%, light gray = 80%, white = 100%. This run was conducted using five cultural features and 10 traits per feature, using the initial conditions shown in Table 1. Each interior site has four neighbors.
Effect of number of features and traits

More features ⇒ homogeneity
More traits ⇒ diversity

**TABLE 2**
Average Number of Stable Regions

<table>
<thead>
<tr>
<th>Number of Cultural Features</th>
<th>Traits per Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>15</td>
<td>1.0</td>
</tr>
</tbody>
</table>

NOTE: These runs were done with a territory of $10 \times 10$ sites, and each interior site had four neighbors. Each condition was run 10 times.
Effect of neighbourhood size

- Three possible number of neighbours per agent (4, 8 and 12)
- **Result**: Larger neighbourhoods result in fewer stable regions
Effect of population size

Figure 2: Average Number of Stable Regions
NOTE: The parameters for these runs are five cultural features, 15 traits per feature, and four neighbors for interior sites. Each territory size was replicated 40 times, except the territories with 50 × 50 sites and 100 × 100 sites territories, which were replicated 10 times.
Problems of the model

Diversity is not persistent if we introduce mutation:
- Klem et al. (2003 b)

Results:
- Small mutation rate ⇒ homogeneity + fluctuations
- Large mutation rate ⇒ unstable diversity

Explanation:
- Mutants ‘bridge’ disconnected regions
Problems of the model (2)

Diversity is not persistent if we introduce noise:

- With small probability $p$, agent $i$ is influenced by agent $j$ no matter how dissimilar they are

Result: Opinion diversity is destroyed.

Explanation: Features adopted by means of the noise allow interaction among disconnected regions
Further extensions of the model

- Metric features (Flache and Macy, 2006)
- The role of homophily and group influence (Flache and Macy, 2007)
- Effect of static social networks (Klemm et al, 2003 a)
- ”Network Homophily” (Centola et al, 2007)
Other interesting models

- Axelrod’s model + Metric features + Bounded confidence (Flache and Macy, 2006), BUT not robust to mutation and noise.

- Negative social influence to explain polarization (Mark, 2003) (Salzarulo 2006), BUT strong assumption and not able to explain consensus and diversity with the same model.

- Persuasive Argument Theory (Mäs and Flache, 2008), BUT not robust to noise.
Summary

1. Consensus, opinion diversity and polarization: Related with key issues in Sociology (from decision-making to innovation, influence of extremism on public opinion, politics ..).

2. 2 models to explain these ‘outcomes’.

3. Still a open topic !?
Bibliography: Texts corresponding to models


Bibliography: Texts proposing extensions of Axelrod’s model


Bibliography: Texts proposing extensions of Axelrod’s model (2)


Bibliography: Other texts


Bibliography: Other texts (2)


- Salzarulo, L. ’A continuous opinions dynamics model based on the principle of meta-contrast’ *Journal of Artificial Societies and Social Simulation* **9**.